

Money and Capital in a Persistent Liquidity Trap*

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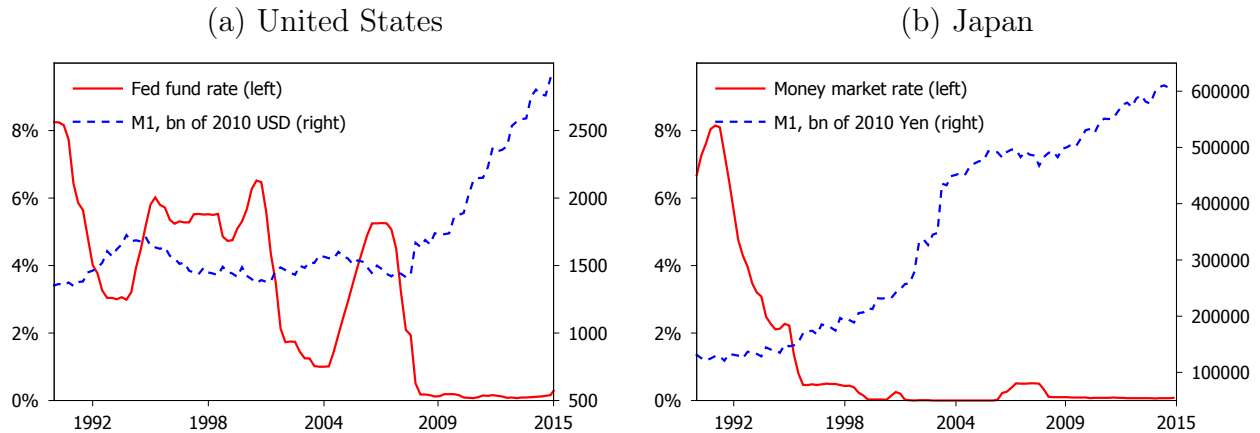
Abstract

In this paper we analyze the implications of a persistent liquidity trap in a monetary model with asset scarcity. We show that a liquidity trap leads to an increase in cash holdings and may be associated with a decline in output in the medium term. This medium-term impact is a supply-side effect that may arise when agents are heterogeneous. It occurs in particular with a persistent deleveraging shock, leading investors to hold cash yielding a low return. Policy implications differ from shorter-run analyses. Quantitative easing leads to a deeper liquidity trap. Exiting the trap by increasing expected inflation or applying negative interest rates does not solve the asset scarcity problem.

Keywords: Zero lower bound, liquidity trap, asset scarcity, deleveraging.

JEL Classification Numbers: E40, E22, E58.

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Source: International Financial Statistics. M1 is deflated by the CPI.

Figure 1: Policy rates and M1 in the US and Japan.

1 Introduction

Periods of persistent liquidity traps typically coincide with substantial increases in cash holdings, as illustrated in Figure 1 for the U.S. and Japan. Moreover, these periods are associated with disappointing levels of investment and of output growth.¹ Can increased money holdings crowd out physical investment and contribute to lower growth? Most macroeconomic models would give a negative answer to this question, since money typically has no long-run effect. In this paper, we argue that in a liquidity trap investment can be negatively related to money holdings. We consider a monetary model with flexible prices, where money is only held for transaction purposes in normal times, with no impact on the allocation of resources. In a liquidity trap, however, investors allocate part of their saving to money holdings that have a low return. With agents heterogeneity, this lower return may then hamper the investment capacity of the economy and have a long-lasting impact on output. This mechanism implies that a liquidity trap may have supply-side effects that contribute to a slower recovery. The policy implications of these supply-side effects differ from shorter-run analyses.

We focus on a liquidity trap generated by a deleveraging shock. Due to this shock, desired aggregate saving exceeds desired investment and the nominal interest rate cannot adjust downward because of the Zero Lower Bound (ZLB). In that case, excess desired saving translates

¹E.g., see International Monetary Fund (2015) for the recent period. Persistently low investment has led to downward revisions of estimates of potential output and fueled speculation as to whether the world economy might be suffering from “secular stagnation”. See Teulings and Baldwin (2015) for an interesting collection of essays on secular stagnation.

into higher real cash holdings. We assume a persistent deleveraging shock, so that the liquidity trap can be persistent, even in the long run after prices have adjusted. This contrasts with the literature focusing on long-run demand effects and modeling long-lasting liquidity traps by assuming persistent nominal rigidities. Instead, we make the conventional assumption that prices are flexible in the long run and analyze the long-term implications of the liquidity trap. This long-term analysis also holds for a transitory, but persistent, deleveraging shock.

More precisely, we introduce money in a model with scarce (liquid) assets due to the lack of income pledgeability, in the spirit of Woodford (1990) and Holmström and Tirole (1998).² Investors find investment opportunities every other period, so that they alternate between investing phases and saving phases. In their investing phase, they use their past liquid saving and borrow to invest, but this borrowing is limited by credit constraints. Agents can save in two liquid assets, real bonds and money. As long as the nominal interest rate is positive, money is dominated as an asset and is held only for transaction purposes. At the ZLB, bonds and money become substitutes and money can be held for saving purposes as well. In this framework, we consider a persistent deleveraging shock, modeled as in Eggertsson and Krugman (2012) by a tightening of the investors' borrowing constraints.³ This shock generates a decrease in the interest rate until the nominal rate eventually hits the ZLB. This creates a gap between the effective real interest rate and the shadow real rate that would prevail without the ZLB. In our model, the fall in the shadow interest rate lasts as long as the deleveraging shock. If this shock is permanent, then the liquidity trap persists in the steady state. The lack of liquid assets in the economy indeed prevents investors from moving away from their credit constraints through saving.

We show that the consequences of a deleveraging shock are very different outside the ZLB and at the ZLB. Outside the ZLB, a deleveraging shock has no effect on capital accumulation and output (in our benchmark specification) as the interest rate can adjust downward and offset the tighter borrowing constraint. However, large deleveraging shocks that bring the economy to the ZLB have a negative effect on capital and output. Since the deleveraging shock reduces

²See also Farhi and Tirole (2012) and Bacchetta and Benhima (2015) for more recent contributions.

³With nominal rigidities, the literature has already shown that such a deleveraging shock can lead to low levels of output and employment in the short run, due to lower demand. Eggertsson and Krugman (2012), Werning (2012), Benigno et al. (2014), or Caballero and Farhi (2015) show this in New-Keynesian models.

the investors' supply of assets, their excess saving is allocated to money in the absence of interest rate adjustment. Money has then two effects on capital accumulation. First, saving in money rather than bonds means that fewer funds are channelled to investment—a negative crowding-out effect. Subsequently, however, money provides funds as it can be liquidated to finance investment—a positive liquidity effect. But since money has a low return, it is a poor source of liquidity, so the crowding-out effect dominates and investment decreases in the long run.

The negative effect of cash in the liquidity trap mainly comes from a Pigou-Patinkin real balance effect, which leads to higher consumption as a share of output and therefore less investment. While real balance effects cannot arise in a Ricardian world, they are present in our framework due to credit constraints and to agent heterogeneity. In addition, for positive inflation rates, the inflation tax also redistributes part of investors' resources to other agents (here, workers), which further hurts investment.

Most of our analysis focuses on steady states, but it is also relevant for transitory, but persistent, deleveraging shocks. We show this by modeling the deleveraging shock as a Markov-switching process, where deleveraging corresponds to the state with a more stringent financial constraint. On impact, the shock has more negative effects than in the long run. After a few years, the economy recovers, but only partially. The economy only recovers completely when the financial constraint parameter comes back to its initial state and the economy gets out of the liquidity trap. When we introduce nominal rigidities—in the form of downward wage rigidity, the impact effect is stronger than in the absence of nominal rigidities, as is well known, but the medium-term effect is similar.

In our analysis, the long-run investment slow-down is due to an increase in *investors'* demand for cash, so that it is crucial that the deleveraging shock affects *investors*.⁴ Indeed, tighter credit constraints among investors increase their net saving. This extra demand for saving is satisfied by money at the ZLB, and their capacity to finance investment is then directly affected by the low return on money. On the contrary, a deleveraging shock affecting only workers has no long-run effects in the liquidity trap, because it does not alter the investors' demand for

⁴Section 1 of the Online Appendix decomposes the rise in cash holdings in the US and shows that it comes from the less constrained firms and households, which would correspond to investors in the model.

saving.⁵ Besides, other types of shocks, such as an increase in the discount rate or a decrease in the growth rate of productivity, do not have a negative long-term effect on the investment rate. In these cases, the crowding-out of investment by cash is compensated by an increase in the aggregate saving rate. Our results therefore suggest that investors' deleveraging is an important factor of growth slowdowns in persistent liquidity traps.

Our framework has different policy implications than traditional shorter-run analyses. In a liquidity trap, typical policies are quantitative easing (QE), negative interest rates, or an increase in expected inflation. These policies may have their merits in the short run, but they have serious drawbacks in the long run. QE operations, by taking public bonds away from the market, decrease the shadow real interest rate and generate a deeper and potentially longer liquidity trap. Negative nominal interest rates or an increase in expected inflation help to exit the liquidity trap by lowering the effective real interest rate. However, these policies do not solve the asset scarcity problem but deteriorate the allocation of resources across time by further lowering the real interest rate. Instead, improving the supply of liquidity helps exiting the liquidity trap by increasing the shadow interest rate. This can be done both through credit easing or through a higher supply of government debt.⁶ However, while a higher supply of liquidity improves the allocation of resources across time, it can have undesirable redistributive effects in some cases by reducing wages. This can happen if investors are net debtors, so that a higher interest rate generates costs that limit their investment capacities and reduces the capital stock.

Our asset-scarce environment is characterized by a low interest rate, so it is prone to rational bubbles. When we allow for bubbles that can be held by savers, we show that they play a role similar to money, generating crowding-out and liquidity effects. By sustaining a higher interest rate, the emergence of a bubble rules out money and brings the economy out of the ZLB.

Related literature The paper is related to the recent literature on persistent ZLB equilibria. In the existing literature, liquidity traps usually arise when the natural rate of interest falls

⁵The empirical literature shows that all sectors of the private economy suffer from deleveraging in the Great Recession. See Mian and Sufi (2010) and Mian and Sufi (2012) for the evidence on households' deleveraging. See Chodorow-Reich (2014), Giroud and Mueller (2015), Bentolila et al. (2009) for the evidence on firms' deleveraging.

⁶Such policies are also discussed in policy circles, e.g., Kocherlakota (2015).

enough to make the nominal rate hit the ZLB (Krugman, 1998; Auerbach and Obstfeld, 2005; Eggertsson and Krugman, 2012; Werning, 2012). In standard models with an unconstrained infinitely-lived representative agent, this cannot be a persistent equilibrium since the natural rate is tightly linked to time preference through the Euler equation. A steady state can only be at the ZLB if inflation is far below target, leaving the real rate and the allocation of resources unchanged, as in the self-fulfilling equilibrium of Benhabib et al. (2001). Schmitt-Grohé and Uribe (2013) add permanent nominal rigidities (a non-vertical long-run Phillips curve) to this framework to get a lower output at the ZLB. Benigno and Fornaro (2015) introduce endogenous growth along permanent nominal rigidities and get a self-fulfilling ZLB steady state with low output, low growth, and a low real interest rate. Moving away from the representative agent framework, Eggertsson and Mehrotra (2014) and Caballero and Farhi (2015) use a non-Ricardian OLG framework with financial frictions, where the equilibrium real rate of interest can be arbitrarily low. Michau (2015) gets a similar feature with wealth in the utility function of an otherwise standard representative agent. Assuming the interest rate is stuck at the ZLB, adjustment in these three papers is supposed to come from a persistently negative output gap, which again requires long-run nominal rigidities. Hence, in the existing literature, stagnation in a persistent liquidity trap remains a demand-side phenomenon.

Like us, Buera and Nicolini (2016), Guerrieri and Lorenzoni (2015) and Ragot (2016) examine the effects of a deleveraging shock at the ZLB in the absence of nominal rigidities. Guerrieri and Lorenzoni (2015) focus on consumer spending in a model where households face borrowing limits, and Ragot (2016) studies optimal monetary policy in a model where money has redistributive effects due to limited participation. In both models, there is no capital accumulation. Closer to our approach, Buera and Nicolini (2016) consider a monetary model where producers need external funds to buy capital. While we focus on the negative relationship between cash holdings and capital, they study the reallocative effects of low real interest rates on total factor productivity and capital, and assume a moneyless economy in most of their paper. Like us, they discuss the trade-offs associated to the inflation policy but do not consider increases in public debt large enough to exit the liquidity trap by raising the shadow interest rate.

The crowding-out and liquidity effects of money we emphasize are reminiscent of the effects of external liquidity in other models where investors' income is not fully pledgeable, such as

Woodford (1990), Holmström and Tirole (1998), and more recently Covas (2006), Angeletos and Panousi (2009), Kiyotaki and Moore (2012), Kocherlakota (2009) and Farhi and Tirole (2012). A short-term crowding-out effect is also present in Andolfatto (2015). The role of money as a saving instrument is also evocative of the literature on the value of fiat money (Samuelson (1958), Townsend (1980)). In our paper, transactions are not constrained by demography or spatial separation, but by the lack of income pledgeability.

Our paper is also related to the literature on bubbles, which are an alternative saving instrument in asset-scarce environments with a low interest rate. In Samuelson (1958) and Tirole (1985), bubble-prone asset-scarcity is due to the OLG structure of their economies. In Martin and Ventura (2012) and Farhi and Tirole (2012), it is due both to the OLG structure and to financial frictions. Asriyan et al. (2016) introduce bubbles in a monetary environment. They also analyse liquidity traps and some of their policy analyses are similar to ours.

The real balance effect that underlies the adjustment mechanism present in our model has been originally studied by Pigou (1943) and Patinkin (1956). More recently, Weil (1991), Ireland (2005), Bénassy (2008) and Devereux (2011) have analyzed real balance effects in OLG models.

The rest of the paper is organized as follows. Section 2 presents the basic model with infinitely-lived entrepreneurs and workers. Section 3 studies the effect of a permanent deleveraging shock in a flexible price steady state before extending the analysis to transitory but persistent shocks with nominal rigidities. Section 4 examines policy options. Section 5 studies several extensions of the benchmark model: workers' deleveraging, bubbles, preference and growth shocks, financial intermediation, inefficient saving technology, idiosyncratic uncertainty, partial capital depreciation, and nominal government bonds. Section 6 concludes.

2 A Model with Scarce Assets and Money

We consider a heterogenous-agents, non-Ricardian monetary model where the supply of bonds and the distribution of money holdings matters. Prices are flexible as we first focus on the long run. In normal times, bonds dominate money and the real interest rate adjusts to balance the supply and demand for bonds. In a liquidity trap, however, bonds and money become perfect

substitutes. The supply and demand of assets are then balanced by an adjustment in real money holdings (coming from either prices or money supply). These two adjustment mechanisms, through interest rates or money holdings, have different implications for investment and output, and therefore for policy. We show that in a liquidity trap real money holdings by investors tend to increase, which may have a negative impact on capital and output in the medium run. This is in particular the case for a deleveraging shock, which we analyze in Section 3. In this section, we describe the model and the equilibrium.

2.1 The Setup

We model a monetary economy with heterogeneous investors, workers, and firms. There are three types of assets: bonds, money, and capital. We assume that bonds are real bonds, that is, promises to pay one unit of final good in the next period.⁷ Denote by r_{t+1} their gross real rate of return expressed in units of final goods: a bond issued in period t is traded against $1/r_{t+1}$ units of final goods. The gross nominal return expressed in units of currency is $i_{t+1} = r_{t+1}E_tP_{t+1}/P_t$, where P_t is the price of the final good in units of currency in period t and E_t denotes the expectation as of time t . While r_{t+1} represents the effective interest rate, at the ZLB we will also consider the *shadow* interest rate r_{t+1}^s , which is the real interest rate that would prevail if the ZLB were not binding.

Money bears no interest; that is, it pays a gross nominal return equal to 1. While bond holdings can be both positive or negative, money holdings are non-negative. In addition, money provides transaction services by relaxing a cash-in-advance constraint faced by workers. In normal times, when the gross nominal return i is strictly larger than 1, money is strictly dominated by bonds as a saving instrument. Then, only workers hold money, for transaction purposes. However, when $i = 1$, a situation that will obtain in a liquidity trap, money becomes as good a saving instrument as bonds and investors start holding money as well.

Investors Following Woodford (1990), investors find investment opportunities every other period, so that they alternate between a saving period and an investment period. This simple alternating approach is a convenient limit case allowing to capture idiosyncratic shocks in a very

⁷The case of nominal bonds is considered in Section 5.

tractable way. Section 5 examines the more general case with idiosyncratic uncertainty on the occurrence of an investment opportunity and shows that the analysis is similar. Consequently, at each point in time there are two groups of investors, investing and saving every other period. We call investors in their saving phase S-investors, or simply savers, and denote them by S , while investors in their investment phase are called I-investors and are denoted by I . Each group is of measure 1. We assume logarithmic utility in order to get closed-form solutions. An individual investor i maximizes

$$U_t^i = E_t \sum_{s=0}^{\infty} \beta^s \log(c_{t+s}^i)$$

where c_t^i refers to her consumption in period t .

In period t , I-investors start with wealth $a_t + \frac{M_t^S}{P_t}$ where a_t and M_t^S are respectively real bond holdings and nominal money holdings inherited from their preceding saving phase. They get an investment opportunity, which consists in a match with a firm. I-investors consume c_t^I , issue b_{t+1} bonds, and invest k_{t+1} in the firm. We abstract from money demand by I-investors, as it is always zero in equilibrium. Their budget constraint is

$$\frac{b_{t+1}}{r_{t+1}} + a_t + \frac{M_t^S}{P_t} = c_t^I + k_{t+1}. \quad (1)$$

In period t , S-investors start with equity k_t and outstanding debt b_t inherited from their preceding investment phase. They receive a dividend $\rho_t k_t$. Then, they consume c_t^S , buy a_{t+1} real bonds and save M_{t+1}^S in money. Their budget constraint is

$$\rho_t k_t = c_t^S + b_t + \frac{a_{t+1}}{r_{t+1}} + \frac{M_{t+1}^S}{P_t}. \quad (2)$$

In general, the return on capital is larger than r_t . Thus, I-investors choose to leverage up when they receive an investment opportunity. But they face a borrowing constraint as they can only pledge a fraction ϕ_t of dividends so that

$$b_{t+1} \leq \phi_t \rho_{t+1} k_{t+1}. \quad (3)$$

In this framework, where investment opportunities are lumpy and investors cannot fully pledge their future income, there is an asynchronicity between the investors' access to and their need for resources. This creates a demand for assets for liquidity purposes in the investors' saving phase.⁸ Both bonds and money can satisfy this demand for liquidity, or demand for assets (we will use these two terms interchangeably). Capital, on the other hand, is illiquid, since it cannot be fully pledged.

Firms There is a unit measure of one-period-lived firms, who are each matched with an I-investor. Firms use their investor's funds to buy capital k_t and produce output y_t with capital and labor through a Cobb-Douglas production function so that $y_t = F(k_t, h_t) = k_t^\alpha h_t^{1-\alpha} + (1 - \delta)k_t$. Labor h_t is paid at real wage w_t and all profits are distributed to I-investors as dividends, i.e., $\Pi_t = y_t - w_t h_t$. As the labor market is competitive, these profits are linear in k and can be rewritten as $\Pi_t = \rho_t k_t$, where ρ is the equilibrium return per unit of capital.⁹ For expositional clarity, we assume full depreciation. The case of partial depreciation $\delta < 1$ is deferred to Section 3.7 of the Online Appendix. In equilibrium, profits are then simply $\rho_t k_t = \alpha y_t$.

Workers There is a unit measure of workers who maximize

$$U_t^w = E_t \sum_{s=0}^{\infty} \beta^s \log(c_{t+s}^w)$$

where c_t^w refers to workers' consumption. They have a fixed unitary labor supply, so that $h_t = 1$ in equilibrium. Their budget constraint is:

$$c_t^w + \frac{M_{t+1}^w}{P_t} + l_t^w = w_t + \frac{T_t^w}{P_t} + \frac{M_t^w}{P_t} + \frac{l_{t+1}^w}{r_{t+1}}, \quad (4)$$

where l^w is the amount of real bonds issued, M^w money holdings, and T^w a monetary transfer from the government.

Workers are subject to a cash-in-advance (CIA) constraint: they cannot consume more than their real money holdings. Assuming the bond market opens before the market for goods, these

⁸We use the term liquidity in the same spirit as Woodford (1990) and Holmström and Tirole (1998).

⁹ ρ is given by $\rho = F(1, 1/k(w)) + 1 - \delta - w/k(w)$ where $k(w)$ is the equilibrium capital-labor ratio defined by $w = F_h(k(w), 1)$.

holdings are the sum of money carried over from the previous period, monetary transfers from the government, and money borrowed on the bond market (net of debt repayment):

$$c_t^w \leq \frac{M_t^w + T_t^w}{P_t} + \frac{l_{t+1}^w}{r_{t+1}} - l_t^w. \quad (5)$$

Workers also face a borrowing constraint

$$l_{t+1}^w \leq \bar{l}_t^w y_{t+1}. \quad (6)$$

We assume that the borrowing limit is linear in the wage bill and therefore proportional to output (since the equilibrium wage bill is a fraction $1 - \alpha$ of output). We allow for the case $\bar{l}^w < 0$, which represents forced saving by workers.

When $\beta r < 1$, which we will assume throughout the analysis, workers would prefer to dissave and always hold the minimum amount of money, so that the CIA is always binding. Together with their budget constraint (4), this implies that their money holdings are simply equal to the wage bill: $M_{t+1}^w/P_t = w_t$. Since the wage bill is equal to $(1 - \alpha)y_t$ in equilibrium, money demand by workers is given by:

$$M_{t+1}^w = (1 - \alpha)P_t y_t. \quad (7)$$

Money supply and government policy Denote by M_t the money supply at the beginning of period t . In period t , the government can finance transfers to agents by creating additional money $M_{t+1} - M_t$ and by issuing real bonds l_{t+1}^g . For simplicity, we assume that the government only makes transfers to workers. The budget constraint of the government is:

$$\frac{M_{t+1}}{P_t} + \frac{l_{t+1}^g}{r_{t+1}} = \frac{M_t}{P_t} + \frac{T_t^w}{P_t} + l_t^g. \quad (8)$$

Several fiscal and monetary policies can be considered. As a benchmark case, we assume that the fiscal authority provides a real supply of bonds that is proportional to output $l_{t+1}^g = \bar{l}_t^g y_{t+1}$ and that the monetary authority controls the growth of money

$$M_{t+1}/M_t = \theta_{t+1}. \quad (9)$$

Transfers to households then adjust to satisfy the budget constraint (8). We assume that money growth is constant in the long run and equal to θ , which enables us to pin down steady-state inflation easily, as it will be equal to θ .

We make the following parametric assumption:

Assumption 1 $\theta > \beta$.

Assumption 1 implies that the economy can only hit the zero lower bound in a steady state where $\beta r < 1$, that is with binding borrowing constraints. Indeed, in the steady state, the nominal gross interest rate is $i = r\theta$. With assumption 1, $i = 1$ implies $\beta r = \beta/\theta < 1$. This assumption is naturally satisfied as long as $\theta \geq 1$, that is with a non-negative steady-state inflation.

Market clearing for bonds and money The market for bonds clears so that

$$b_{t+1} + l_{t+1}^w + l_{t+1}^g = a_{t+1}. \quad (10)$$

Similarly, equilibrium on the money market is given by:

$$M_{t+1}^S + M_{t+1}^w = M_{t+1}. \quad (11)$$

Sequences of leverage We assume that the sequences of leverage $\{\phi_t, \bar{l}_t^w, \bar{l}_t^g\}$ are exogenous and deterministic. As a consequence, investors have perfect foresight, which will enable us to derive closed-form solutions.

2.2 Equilibrium

In an asset-scarce environment, the dynamics of the economy can be summarized by four key equations: a complementary slackness condition that determines whether the economy is in a liquidity trap or not, the Euler equation for savers, the investors' aggregate budget constraint and the equilibrium on the money market.

Asset scarcity and binding borrowing constraints We focus on equilibria where strong borrowing constraints prevent borrowers from supplying the saving instruments needed by savers. In such an “asset-scarce” economy, we will have $\beta r < 1$ in the long run, so the borrowing constraints are binding for workers and I-investors at the vicinity of the steady state, which we assume throughout.

A binding borrowing constraint for workers sets their supply of assets to $l_{t+1}^w = \bar{l}_t^w y_{t+1}$. We define the supply of bonds to investors by the rest of the economy, which includes workers and the government, by

$$l_{t+1} = l_{t+1}^w + l_{t+1}^g = \bar{l}_t y_{t+1} \quad (12)$$

where $\bar{l} = \bar{l}^g + \bar{l}^w$. In equilibrium, l_{t+1} is also the net position of investors.

The zero lower bound and money demand The portfolio choice of S-investors can be summarized by the following complementary slackness condition:

$$M_{t+1}^S \left(r_{t+1} - \frac{P_t}{P_{t+1}} \right) = 0. \quad (13)$$

As long as $i > 1$, money has a strictly lower expected return than bonds and investors hold the minimum amount of money, which is zero. Then, we have $M^S = 0$. We refer to periods where $i > 1$ and investors hold no money as “normal” periods.

When $i = 1$, that is $r_{t+1} = P_t/P_{t+1}$, bonds and money become perfect substitutes for savers, and they start holding money for saving purposes, so $M^S \geq 0$. We refer to periods where $i = 1$ and S-investors hold money as “liquidity trap” periods.

Euler equation of savers S-investors are typically unconstrained, so their Euler equation is satisfied: $1/c_t^S = \beta r_{t+1}/c_{t+1}^I$. With log-utility, consumption is a fraction $1 - \beta$ of wealth for both types of investors.¹⁰ Then, $c_{t+1}^I = (1 - \beta)(a_{t+1} + M_{t+1}^S/P_{t+1})$ and $c_t^S = (1 - \beta)(\rho_t k_t - b_t) = (1 - \beta)(\alpha y_t - b_t)$. Assuming binding borrowing constraints (3) and (6), and using the market

¹⁰The proof of this property is available upon request. The case of log-utility is a realistic one when it comes to modeling the saving behavior of agents, as a unitary elasticity of intertemporal substitution is well within the estimated ranges.

clearing condition for bonds (10), the Euler equation of S-investors can be rewritten

$$\beta\alpha(1 - \phi_{t-1})y_t = \frac{1}{r_{t+1}} [(\phi_t\alpha + \bar{l}_t)y_{t+1} + m_{t+1}^S]. \quad (14)$$

where $m_{t+1}^S = M_{t+1}^S/P_{t+1}$ are the real money holdings of S-investors. This Euler equation can also be interpreted as an equilibrium condition for saving instruments. The left-hand side (LHS) is the demand for saving instruments by S-investors, which depends on current income. The right-hand-side (RHS) is the supply of saving instruments. The first term is the supply of bonds, which depends on future pledgeable income. It depends on ϕ , the leverage ratio of I-investors, and on \bar{l} , the leverage ratio of workers and the government. Finally, the last term on the RHS corresponds to money used by S-investors as a saving instrument.

Aggregate budget constraint Substituting for consumption, the budget constraints of I-investors and S-investors (1) and (2) become respectively $\beta(a_t + M_t^S/P_t) = k_{t+1} - b_{t+1}/r_{t+1}$ and $\beta(\alpha y_t - b_t) = M_{t+1}^S/P_t + a_{t+1}/r_{t+1}$. Aggregating these two constraints and using the bond market clearing condition (10), we find

$$k_{t+1} + \pi_{t+1}m_{t+1}^S + \frac{1}{r_{t+1}}\bar{l}_ty_{t+1} = \beta [(\alpha + \bar{l}_{t-1})y_t + m_t^S] \quad (15)$$

where $\pi_{t+1} = P_{t+1}/P_t$ is the gross rate of inflation. This equation represents the aggregate resource constraint of S- and I-investors, and describes capital accumulation. Aggregate saving (on the RHS) must be equal to aggregate investment in capital, bonds and money (on the LHS).

Consider how money affects capital accumulation. First, on the LHS, an increase in desired money holdings by S-investors decreases the capital stock, because other things equal the corresponding funds are not channeled to I-investors. This is the *crowding-out effect of money*. For a given level of future real money holdings that the S-investors want to secure, the crowding-out effect is stronger if inflation, which is the price of (real) money, is larger. Second, on the RHS, past saving in money of current I-investors increases the capital stock, because it can be liquidated to finance investment. This is the *liquidity effect of money*. This liquidity effect is stronger if β is larger, because then I-investors use a higher share of their wealth to invest.

Note that the bond's external position of investors has similar effects, except that the price of liquidity in the case of bonds is not inflation but $1/r_{t+1}$.

Importantly, this equation shows that I-investors' leverage ϕ does not matter outside its potential equilibrium effect on the interest rate, out of the liquidity trap, or through its effect on the demand for money in the liquidity trap. This is because the net position of investors as a whole, $a - b$, ultimately depends on the net supply of bonds by the rest of the economy $l = \bar{l}y$, as $a - b = l$ from (10). This is an important result that greatly simplifies the analysis. To understand the dynamics of capital, it is enough to study the crowding-out and liquidity effects of \bar{l} and m^S .

Money market Substituting (7) into (11), we get

$$\frac{M_{t+1}}{P_t} = (1 - \alpha)y_t + \pi_{t+1}m_{t+1}^S. \quad (16)$$

Money supply has to be equal to the demand for money for transaction purposes plus the demand for money for saving purposes. With perfectly flexible prices, this equation ensures that any real demand for money can be met through a price adjustment, even with a predetermined money supply M_{t+1} .

Equilibrium The Euler equation (14), the aggregate resource constraint (15), and the money market equilibrium (16) describe a constrained equilibrium, which can be formally defined in the following way:

Definition 1 (Constrained equilibrium) *Consider an exogenous sequence of leverage $\{\phi_t, \bar{l}_t^w\}_{t \geq 0}$, a policy $\{\theta_{t+1}, T_t^w, \bar{l}_t^g\}_{t \geq 0}$ satisfying (8), and initial assets $\{k_0, M_0, M_0^S, M_0^w\}$. The associated constrained equilibrium is an allocation $\{y_t, k_{t+1}, M_{t+1}, M_{t+1}^w, M_{t+1}^S, m_{t+1}^s, \bar{l}_{t+1}\}_{t \geq 0}$ and a price vector $\{i_{t+1}, r_{t+1}, w_t, P_t, \pi_{t+1}\}_{t \geq 0}$ satisfying $\pi_{t+1} = P_{t+1}/P_t$, $i_{t+1} = r_{t+1}\pi_{t+1}$, $\bar{l}_{t+1} = \bar{l}_{t+1}^w + \bar{l}_{t+1}^g$, $m_t^S = M_t^S/P$, $y_t = k_t^\alpha$, $w_t = (1 - \alpha)y_t$, (7), (9), (13), (14), (15), and (16).*

In the next section, we first focus on steady state equilibria. It will be useful to distinguish between normal and liquidity-trap steady states. The definition of these steady states is made formally in the following definition:

Definition 2 (normal and liquidity-trap steady states) *A constrained steady state is a constrained equilibrium where $\{\phi, \bar{l}^w, \theta, \tau^w, \bar{l}^g, y, k, m, m^w, m^S, \bar{l}, i, r, w, \pi\}$ are constant, where $\tau^w = T^w/P$, $m = M/P$, and $m^w = M^w/P$. A normal steady state is a constrained steady state satisfying $i > 1$ and $m^S = 0$. A liquidity-trap steady state is a constrained steady state satisfying $i = 1$ and $m^S > 0$.*

3 The Impact of Investors' Deleveraging

This section studies the effects of deleveraging. In our setting, a deleveraging shock on investors can be modeled by a drop in ϕ . We first consider permanent shocks, which allows us to analyze changes in steady states. Then we simulate the effects of a persistent deleveraging shock. In that case, we will also introduce nominal rigidities in the form of downward nominal wage rigidities to study transition dynamics.

3.1 Long-run impact of Permanent Deleveraging

A deleveraging shock leads to an excess net demand for saving instruments by investors. The equilibrium implications of this excess demand are very different depending on whether the economy is in a normal equilibrium or in a liquidity trap. In normal equilibria, adjustment comes from a lower equilibrium interest rate which helps restore a higher supply of bonds. In the liquidity trap, as the interest rate cannot adjust, the higher net demand for saving instruments by investors takes the form of higher money holdings. As we will see, this diverts resources away from investment and leads to lower capital and output in the long-run.

In the following, we first focus on the case $\bar{l} = 0$ where investors are in autarky: S-investors lend to I-investors. In addition to being simpler, this is also a realistic description of the US prior to the crisis: we show in Section 2.1 of the Online Appendix that the net position in financial assets of non-financial corporate businesses was indeed close to 0 in the years 2000 prior to the crisis. Afterwards, we briefly describe how the analysis would change with $\bar{l} \neq 0$.

Autarkic investors As capital accumulation does not depend directly on ϕ , neither does the long-run capital stock. When $\bar{l} = 0$, the dynamics of capital accumulation in a normal

equilibrium, given by (15) with $m^S = 0$, is also independent of the real interest rate r , so the capital stock does not depend at all on ϕ , neither directly nor indirectly through its effect on the interest rate. The capital stock is indeed given by:

$$k = \beta\alpha y = \beta\alpha k^\alpha. \quad (17)$$

A deleveraging shock on investors (a decrease in ϕ) affects the distribution of wealth between S- and I-investors, but not their aggregate saving, so it leaves the capital stock unchanged. This requires a change in the interest rate as an equilibrating mechanism. Indeed, for a given interest rate, the shock generates a decrease in the bond supply b by I-investors. Besides, as S-investors start the period with less debt, it increases their wealth and hence their demand for bonds a . Since the net supply of bonds by the rest of the economy remains unchanged at zero, the adjustment takes place through a decrease in the interest rate, which enables I-investors to borrow more. This is clear from the Euler equation (14), which defines r in the normal steady state as

$$r = \frac{\phi}{\beta(1 - \phi)}. \quad (18)$$

Notice that a decrease in r implies a proportional decrease in $i = r\theta$ for a given steady-state inflation rate θ . Therefore, a strong contraction of credit may lead to the ZLB. This is the case when $\phi/[\beta(1 - \phi)] \leq 1/\theta$. Similarly, a high enough ϕ brings the equilibrium interest rate at $1/\beta$. Beyond this, the credit constraint is not binding anymore.

If i hits the ZLB at $i = 1$, the equilibrium becomes a liquidity trap. The effective real interest rate is simply $1/\theta$. We define the shadow real interest rate r^s as the interest rate that would prevail if the ZLB were not binding. It is given by the right-hand side of (18), i.e., $r^s = \phi/[\beta(1 - \phi)]$.¹¹ We then define the interest rate gap as the difference between the effective and the shadow interest rates:

$$\Delta \equiv r - r^s = \frac{1}{\theta} - \frac{\phi}{\beta(1 - \phi)}$$

¹¹The shadow rate goes to 0 when ϕ goes to 0. This is an extreme situation where savers, absent money, would have no instruments to trade intertemporally. Section 5 introduces an alternative inefficient saving technology, which puts a strictly positive lower bound on the shadow rate.

We think of the magnitude of this gap as the depth of the liquidity trap.

In a liquidity trap steady state, the Euler equation (14) becomes:

$$m^S = \alpha \left[(1 - \phi) \frac{\beta}{\theta} - \phi \right] y. \quad (19)$$

m^S/y is decreasing in ϕ : an increase in investors' net demand for saving instruments triggered by a deleveraging shock is now accommodated by an increase in their real money holdings m^S . Indeed, at the ZLB, bonds and money have the same return and money becomes a saving instrument. It is also interesting to notice that m^S is proportional to the interest rate gap Δ :

$$m^S = \kappa \Delta y \quad (20)$$

where $\kappa = \alpha\beta(1 - \phi)$. The magnitude of investors' real money demand is therefore also a measure of the depth of the liquidity trap.

This switch to money takes out resources from investment, as suggested by (15), which becomes in a steady state

$$k = \beta\alpha y - (\theta - \beta)m^S. \quad (21)$$

From Assumption 1, we have $\theta > \beta$ and holding additional money entails a net resource cost that decreases the long-run stock of capital. Indeed, in the steady state, the cost of saving in money for S-investors, $\pi_{t+1} = \theta$, is then larger than the I-investors' propensity to use money holdings for investment β . The reduction in the funds coming from S-investors is therefore not compensated by the liquidity service of money to I-investors. In other words, the crowding-out effect of money overcomes its liquidity effect.

Notice that asset scarcity is crucial here. First, it generates a persistent drop in interest rate, making the liquidity trap persistent. Second, asset scarcity means that the return on bonds, and hence the return on money in the liquidity trap, is below $1/\beta$, so bond or money accumulation in the liquidity trap is costly.

The net resource cost for investors arises because of a real balance effect together with an

inflation tax, as can be seen by rewriting Equation (21):

$$k = \beta\alpha y - \underbrace{(\theta - 1)m^S}_{\text{Inflation tax}} - \underbrace{(1 - \beta)m^S}_{\text{Extra consumption}} .$$

Because cash is considered as net wealth by investors (a consequence of the non-Ricardian structure of the model), they consume a fraction $1 - \beta$ of it. In addition, a fraction $\theta - 1$ of cash is lost as an inflation tax, which is redistributed to workers through transfers.¹²

How does the adjustment in investors' real money holdings m^S take place? From (16) taken in the steady state, we have $m = M/P = (1 - \alpha)y/\theta + m^S$. Since workers' money holdings always equal their wage bill, the supply of total real money holdings m has to increase. For a given path of money supply, given by (9), this implies a downward shift in the path of prices P_t . At the ZLB, a deleveraging shock is disinflationary, which endogenously increases real money holdings to accommodate the higher net demand for saving instruments by investors. This is intuitive: a larger demand for saving instruments bids up the price of bonds out of the ZLB, i.e. decreases the interest rate, and bids up the price of money at the ZLB, i.e. decreases the price level.

Using this analysis, we establish the following Proposition:

Proposition 1 (Steady state with autarkic investors) *Define $\phi_T = \beta/(\theta + \beta)$ and $\phi_{max} = 1/2$. If $0 < \phi < \phi_{max}$, then there exists a locally constrained steady state with $r < 1/\beta$.*

- (i) *If, additionally, $\phi \geq \phi_T$, then the steady state is normal.*
- (ii) *If $\phi < \phi_T$, then the steady state is a liquidity trap.*
- (iii) *In the normal steady state, the real interest rate r and the nominal interest rate i are increasing in ϕ , $m^S = 0$ and k is invariant in ϕ .*
- (iv) *In the liquidity-trap steady state, the real interest rate r is invariant in ϕ , m^S/y is decreasing in ϕ and k is increasing in ϕ .*

Proof. See Appendix A. ■

¹²This second effect would be lower if investors also received transfers from the government.

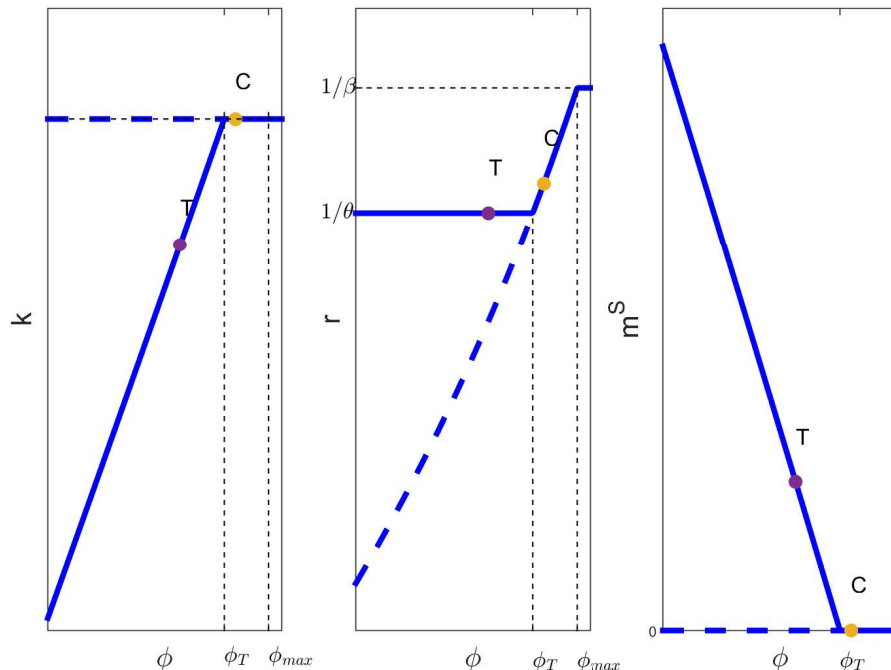


Figure 2: Steady states - Comparative statics w.r.t. ϕ , with $\bar{l} = 0$

This Proposition establishes under which condition on ϕ the steady state is normal or a liquidity trap. It is illustrated in Figure 2. The solid lines show the levels of k , r , and m^S as a function of ϕ , while the broken lines show the levels of the shadow rate r^s and of k and m^S if the ZLB were not binding. For intermediate values of ϕ (between ϕ_T and ϕ_{max}), the normal real interest rate r is higher than $1/\theta$, and the steady state is normal as the nominal interest rate i is above the ZLB, as is illustrated by equilibrium C . When ϕ falls below ϕ_T , the steady state becomes a liquidity trap where the effective interest rate is $r = 1/\theta$ and is larger than the shadow rate r^s . It is characterized by positive real money holdings among investors, for saving purposes, as illustrated by point T .

As long as the economy is in the normal steady state (when $\phi > \phi_T$), a permanent deleveraging shock on investors (a decrease in ϕ) has no effect on capital, but it has a negative effect on the real interest rate r , as illustrated by Figure 2. But a deleveraging shock large enough to make the economy fall into a liquidity trap (by bringing ϕ below ϕ_T), has negative long-run effects on capital and output. A permanent deleveraging shock, as the one that brings the economy from C to T in the figure, is then consistent with a lower output. The effects come from the disinvestment due to the resource cost of money, thus from the supply side of

the economy, and hold in the absence of any nominal rigidity. This contrasts with the recent literature, where long-run stagnation is driven by a fall in consumption demand in the presence of persistent nominal rigidities.

The fact that higher money holdings lead to lower capital and output in the long run does not imply that investors would be better off if money did not exist. By putting a lower bound on the real rate of interest, money helps investors better smooth consumption across time. Under a mild assumption on the degree of decreasing returns to scale to capital, α , this can be shown to make both groups of investors better off in a liquidity trap steady state than they would be in the corresponding normal steady state, despite the lower capital stock (see Section 4.1 of the Online Appendix). Workers may however be hurt by lower wages.

Investors are net debtors We now turn to the general case where investors have a non-zero net position. In particular, we have in mind the case of investors as net debtors ($\bar{l} < 0$), which corresponds to the situation in the US until the nineties. We only briefly sketch the analysis here, and defer a full treatment to Section 3.1 of the Online Appendix.

In the normal steady state, the capital accumulation equation and the Euler equation become

$$k = \beta\alpha y - \left(\frac{1}{r} - \beta\right)\bar{l}y, \quad (22)$$

$$r = \frac{\phi + \bar{l}/\alpha}{\beta(1 - \phi)}. \quad (23)$$

There are two differences with the autarky case. First, in (23), the interest rate is now increasing in \bar{l} , as the latter measures the supply of bonds by the rest of the economy. Second, in (22), the interest rate now has a redistributive effect between investors and workers, which affects capital accumulation. The sign of the effect depends on the sign of \bar{l} : when $\bar{l} < 0$, a lower interest rate reduces the cost of debt and allows investors to accumulate more capital. This implies that a deleveraging shock actually *increases* the long-run capital stock in the normal economy.¹³

In a liquidity trap however, a deleveraging shock still has a negative long-run effect on

¹³The positive effects on capital accumulation of financial frictions is not an uncommon result: uninsurable risk and credit constraints in Bewley-Aiyagari models notoriously leads to an over-accumulation of capital. See Aiyagari (1994), Krusell and Smith (1997), Covas (2006), and Dávila et al. (2012).

capital. In that case, as money and bonds are perfect substitutes, capital accumulation is not affected by the net supply of bonds \bar{l} per se, but by the total amount of net liquidity $s = m^S + \bar{l}y$, which plays the same role as cash holdings in the autarky case. The capital accumulation and the Euler equations become:

$$k = \beta\alpha y - (\theta - \beta)s, \quad (24)$$

$$s = \alpha \left[(1 - \phi) \frac{\beta}{\theta} - \phi \right] y. \quad (25)$$

Notice that we still have $m^S = \kappa\Delta y$, where the shadow rate is now defined by the right-hand side of (23). We therefore refer to $\bar{l}y$ as shadow liquidity, since $s = \bar{l}y$ when $\Delta = 0$.

3.2 Impact of Transitory Deleveraging

Steady state comparisons are helpful to derive closed-form solutions and facilitates the analysis, but they imply a permanent liquidity trap, arguably a non-realistic feature, and abstract from transition dynamics. We now turn to transition dynamics following a transitory deleveraging shock. To do so, we model the leverage parameter ϕ as a Markov-switching process taking two values: ϕ^H corresponding to normal times, and ϕ^L corresponding to the liquidity trap. The probabilities of falling into and exiting the trap are assumed to be low enough so that we can neglect them in the simulations below.

Since we have focused on the long term so far, we have assumed flexible prices. In order to discuss transition dynamics in a meaningful way, we introduce nominal frictions with downward wage rigidities, in the spirit of Schmitt-Grohe and Uribe (2011). We suppose that the nominal wage, defined as $W_t = P_t w_t$, cannot decrease too much from period to period:

$$W_t = \max \{ \gamma W_{t-1}, W_t^* \} \quad (26)$$

where $\gamma \in (0, \theta)$ is the degree of nominal rigidities and W^* is the nominal wage that would satisfy full employment: $W_t^* = p_t(1 - \alpha)k_t^\alpha$. If $W_t^* \geq \gamma W_{t-1}$ then wages can adjust and there is full employment. Otherwise, there is some unemployment: $h_t < 1$, where the level of

employment h_t is determined by

$$\gamma W_{t-1} = p_t(1 - \alpha) \left(\frac{k_t}{h_t} \right)^\alpha.$$

These rigidities are not active in the steady state where prices grow at rate θ , which is by assumption larger than γ , so our steady state analysis is valid in this framework. But nominal rigidities can affect the short-term adjustment to a deleveraging shock.

With nominal rigidities, a deleveraging shock large enough to move the economy to the ZLB creates a negative output gap in the short run, as in the existing New Keynesian literature. The intuition is best described by Equation (16), the market-clearing condition for money: $M_{t+1} = (1 - \alpha)P_t y_t + M_{t+1}^S$. When the economy hits the ZLB, money demand by investors M^S increases. If the monetary authority does not react, adjustment has to come from a lower nominal output $P_t y_t$. If prices cannot adjust quickly, adjustment in the short run requires a drop in output. In our framework, this takes place through the labor margin, which will not be at full employment.

Calibration The model is calibrated to fit the recent experience of the US at the ZLB. The time period is defined to be a year. We calibrate the balance sheet parameters \bar{l}^g and \bar{l}^w to match their empirical counterparts in the US in 2006. We show in Section 2.2 of the Online Appendix that the net position of the general government and the monetary authority in interest-bearing assets was about 40% of GDP. However, this supply of assets was not available to the rest of the economy: it was used as saving instruments by the rest of the world, whose net position in these instruments was about -40% of US GDP. The net supply available to the domestic economy is thus approximately 0. Together with the assumption of autarkic entrepreneurs, this implies $\bar{l}^g = \bar{l}^w = 0$.

The factor of time preference β is set to 0.96 and ϕ^H to 0.495 in order to match a real interest rate of 2%, consistent with the 10-year TIPS before the crisis, and a real rate of return on capital of 4% which implies a realistic 200 bp corporate spread. We make conventional choices for the capital share $\alpha = 0.33$, the depreciation rate $\delta = 0.10$, and we set $\theta = 1.02$ to get a steady state inflation of 2%.

Finally, to discipline the choice of ϕ^L , which gives the extent of deleveraging, and the degree

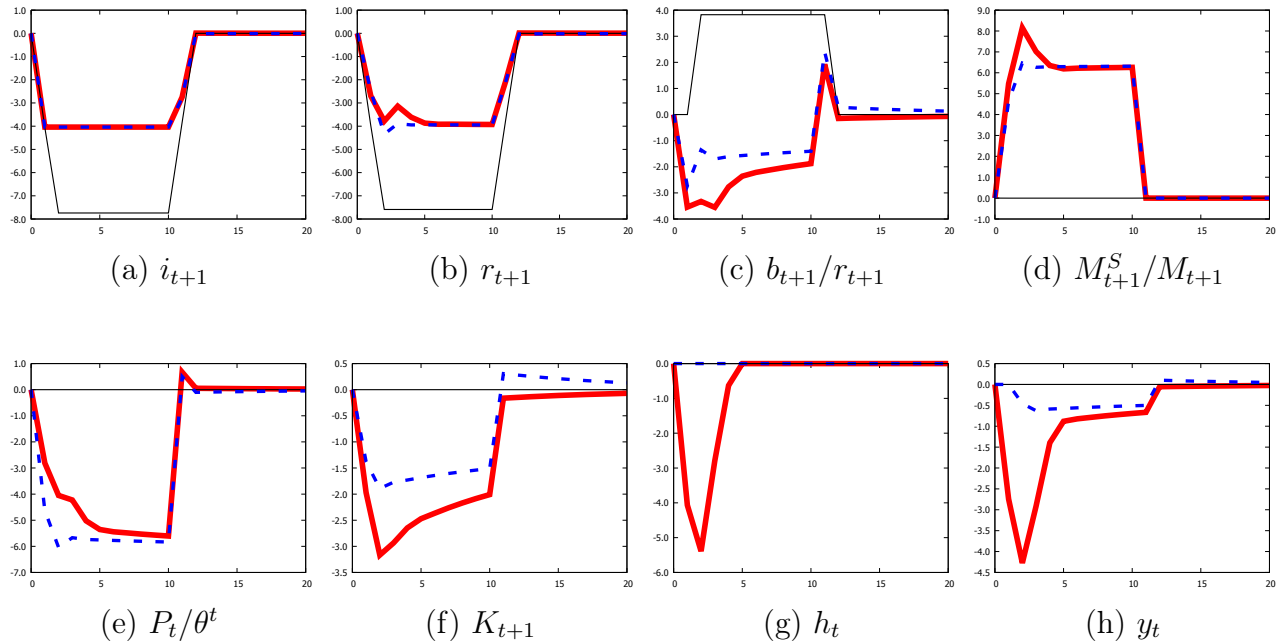


Figure 3: Transitory dynamics after a deleveraging shock. The shock hits in period 1 and lasts for 10 years. Thick red line: downwardly-rigid wages. Dashed blue line: flexible wages. Thin black line: flexible wages and no ZLB. All variables are in relative deviation from initial steady state, in percent, except interest rates and M^s/M which are in absolute deviation from initial steady state, in percent.

of nominal rigidity γ , which drives the increase in unemployment, we match the response of investment and unemployment during the crisis in the US. We set $\phi^L = (1 - 0.039)\phi^H$ and $\gamma = 1.005$ to reproduce the 20% peak-to-trough variation of non-residential investment and the 5.5 pp increase in civilian unemployment of the data.¹⁴

Results The impact of a transitory deleveraging shock is shown in Figure 3. The shock hits in period 1 and is reversed after 10 years in period 11. The dashed blue line represents the baseline case without nominal rigidities. The thin black solid line represents the outcome in the absence of zero lower bound, for comparison purposes. The drop in ϕ generates both a drop in the supply of and a rise in the demand for assets by investors. In the absence of zero lower bound, the real interest rate accommodates this excess demand for assets by dropping substantially (panels a and b). The large decrease in real rate compensates the tightened financial constraint, allowing borrowing to increase (panel c) and accommodate higher saving.

¹⁴Our calibration of γ implies a 0.5% lower bound on wage inflation. With 2% steady-state inflation, this implies that real wages downwardly adjust by at most 1.5% per year.

With autarkic investors, capital and output are unaffected. In contrast, with the zero lower bound, the nominal interest rate is stuck at its lower bound for an extended period of time, which prevents the real rate from adjusting. The excess demand for saving by investors is then channeled to money: M^S increases (panel d). The increase in the demand for money is perfectly accommodated by the fall in prices (panel e). Because the decrease in real rate does not compensate the tightening of the financial constraint, borrowing decreases (panel c) and the capital stock drops on impact (panel f).

After the initial drop, the capital stock recovers, but does not go back to its initial value. As long as the economy stays in the liquidity trap, it remains persistently low. This medium-run effect corresponds to the effect of deleveraging due to the cost of liquidity highlighted in the steady state analysis. Output shows the same pattern as capital: an initial drop followed by a capped recovery (panel h).

Consider now the thick red line, which is drawn under the assumption of downward wage rigidity. This rigidity prevents the decrease in nominal wage needed to clear the labor market given the lower price level. As a result, there is involuntary unemployment, which amplifies the drop in output. It also worsens the financing capacities of investors further and hence lowers the capital stock even more. On impact, labor, output and capital are thus more strongly hit than with flexible wages (panels f, g, and h). These demand effects are very strong, but they only take place in the short run. As time goes by, real wages slowly adjust and all variables converge toward their level under flexible wages.

This analysis shows that the short-run impact of deleveraging is stronger than in the medium run, and even more so in the presence of nominal rigidities. But in the medium run, the effects caused by the scarcity of assets prevail. Contrary to the New Keynesian literature, the economy lingers at the ZLB with a lower capital stock and a lower level of output, even after wages have adjusted and the output gap has closed. In the medium run, steady-state effects prevail. We therefore focus mostly on steady state analysis in the remainder of the paper.

We also show in Section 3.4 of the Online Appendix that a monetary expansion taking the form of transfers to workers (the so-called “helicopter money”) is able to almost replicate the flexible wage equilibrium. While this policy has no effect at the ZLB in standard models,¹⁵ it is

¹⁵In Krugman (1998), for instance, money creation taking the form of transfers has no effect at the ZLB with

efficient in our non-ricardian framework. By increasing money supply, the monetary authority accommodates the demand for money holding by investors, making it unnecessary to decrease output or the price level. Monetary policy can then be very potent to mitigate the short-run impact of deleveraging, but is unable to address its medium-run impact (unless it changes the inflation target itself, see below).

4 Policy

We examine the policy implications of the model in exiting the liquidity trap. We consider standard policies: public debt issuance (including implications for quantitative and credit easing), negative interest rate on money, inflation and fiscal policy. Previous studies focus on short-term effects in the presence of nominal rigidities and hence demand-side policies are paramount. Our analysis instead highlights long-run effects that arise independently from nominal rigidities and therefore put emphasis on supply-side effects. How the effects of such policies translate to a welfare analysis is of course not straightforward in our framework with heterogeneous agents. Nevertheless, the supply of liquidity is a key factor outside of the ZLB.

Exiting from a liquidity trap implies driving the interest rate gap to zero. The authorities can eliminate the interest rate gap either by decreasing the effective rate or by increasing the shadow rate. We have:

$$\Delta = \frac{i}{\theta} - \frac{\phi + \bar{l}/\alpha}{\beta(1 - \phi)}$$

While a strict ZLB implies $i = 1$, we can allow $i < 1$ to analyze the impact of negative interest rates.

In this section, we examine the various policies that can eliminate the interest rate gap, assuming a constant ϕ . The government can choose the growth rate of money θ , its debt/GDP ratio \bar{l}^g or its primary deficit τ^w (equal to lump-sum transfers on workers). However, these three variables cannot be chosen independently as they are linked by the government budget constraint:

$$(\theta - 1)m + \bar{l}^g y \left(\frac{1}{r} - 1 \right) = \tau^w \tag{27}$$

pre-set prices (see footnote 11 of this work).

The first term on the left-hand side is seigniorage and the second term is related to debt service. The real value of money m and output y are determined by the private economy. We first characterize policy by (θ, \bar{l}^g) and let τ^w adjust to balance the budget constraint. At the end of the section we examine the constraints on fiscal policy τ^w .

4.1 Enhancing Shadow Liquidity

In an environment with scarce assets, the public supply of liquidity plays a crucial role. At the ZLB though, public debt only affects the shadow interest rate, as money also plays the role of liquidity. However, by increasing public debt, which is shadow liquidity at the ZLB, the government can increase the shadow interest rate and help the economy exit the ZLB.

Public Debt and the ZLB An increase in the supply of government bonds, by increasing \bar{l} , can obviously bring the economy out of the liquidity trap by increasing the shadow interest rate and shadow liquidity. However, marginal changes in \bar{l} only affect shadow values as long as the economy remains in the liquidity trap, consistently with the “irrelevance result” highlighted in the literature. Indeed, the private demand for liquidity s is fixed at the ZLB, as shown by Equation (25). Within liquid assets, money and bonds are substitutes, so an increase in the supply of bonds is matched by a lower demand for real money balances. To accommodate for lower real money balances, prices increase, unless the central bank intervenes to stabilize prices by decreasing money supply. Only a massive increase in public debt, that fully compensates for the private deleveraging shock, can bring the economy out of a liquidity trap.

Section 4.2 of the Online Appendix shows that an adequate supply of liquidity even enables the economy to reach a Pareto-*efficient* equilibrium. Indeed, by raising the real interest rate, this enables optimal consumption smoothing by all agents as well as the optimal level of capital (see Proposition 9 in the Online Appendix).¹⁶ However, as we will see, this does not imply that transition dynamics are Pareto-efficient, nor that the new long term equilibrium Pareto-*improves* on the initial one.

¹⁶The proposition shows that the efficient level of capital is given by $k = \beta\alpha y$. From Equation (22), this level obtains when $r = 1/\beta$, which also corresponds to perfect consumption smoothing, and requires a high enough public debt $\bar{l} = \bar{l}_{max}(\phi)$. In the case where investors are net debtors out of the ZLB, capital is too high compared to a Pareto-efficient allocation and a higher public debt crowds out this inefficiently high capital stock.

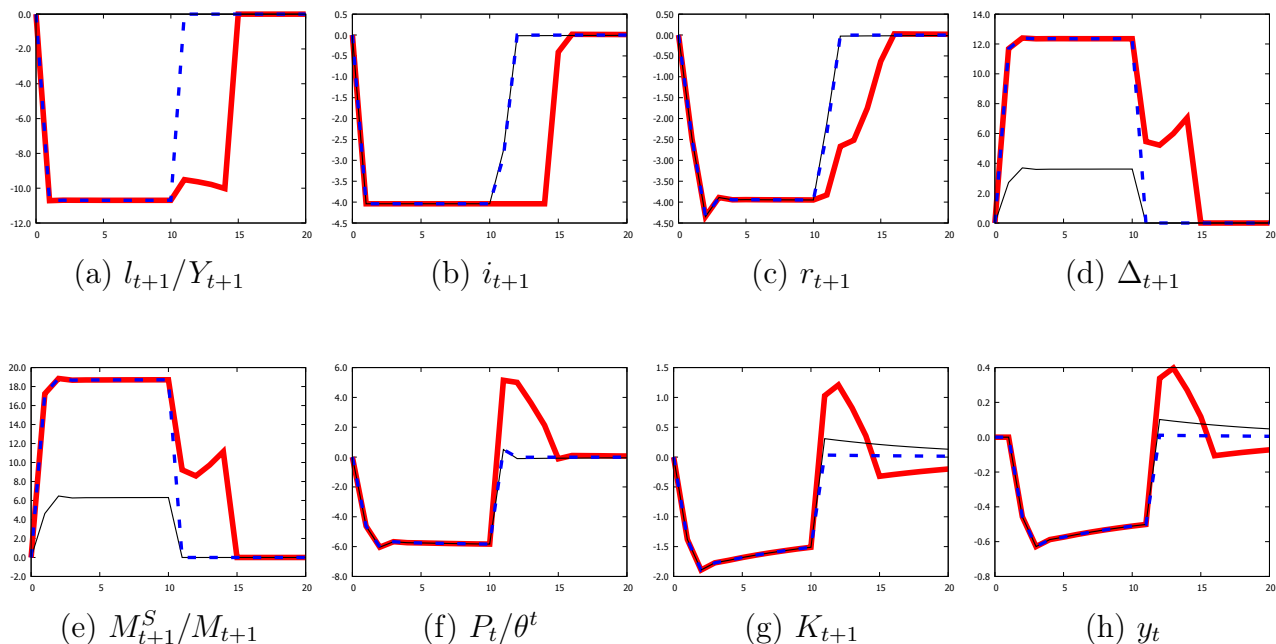


Figure 4: Transitory dynamics after an unexpected deleveraging shock. The shock hits in period 1 and lasts for 10 years. Thick red line: quantitative easing with late exit. Dashed blue line: quantitative easing with early exit. Thin black line: no quantitative easing. All variables are relative deviation from initial steady state, in percent, except interest rates, l/Y and M^s/M which are in absolute deviation from initial steady state, in percent.

Quantitative Easing The above analysis implies that QE has no effect per se in the liquidity trap steady state. QE consists in creating money through open market operations, i.e., increasing M by decreasing Pl^g . Since money and government bonds are perfect substitutes, this has no effect in our setting.¹⁷ However, QE entails a decrease in the available amount of government bonds l^g , which decreases shadow liquidity and the shadow interest rate. QE therefore leads to a deeper liquidity trap. If the central bank does not want to linger in a liquidity trap, it is thus important to time the exit from QE appropriately.

Figure 4 illustrates the effect of QE with a late or early exit. As in Figure 3, a deleveraging shock hits the economy in period 1 and stops in period 11. Wages are assumed to be flexible. We suppose the central bank implements QE by buying bonds worth 10% of GDP when the shock hits in period 1.¹⁸ The dashed blue line displays the case of early exit where the central

¹⁷Note that we abstract from some potential channels of QE. In particular, the perfect substitutability of money and bonds means that there is no broad portfolio balance channel that could lower term or risk premia. See Borio and Disyatat (2009) for a detailed description of the channels of QE.

¹⁸According to H.4.1 Federal Reserve statistical releases, the large-scale asset purchase programs of 2010–2014, usually referred to as QE2 and QE3, increased the amount of securities held at the Federal Reserve by 9 percent of GDP. The total increase since 2006 amounts to 17 percent of GDP.

bank sells the bonds in period 11 when deleveraging stops. The thick red line represents the case of late exit where the central bank announces in period 11 that it will hold the bonds for four additional years, and does so. As a benchmark, the thin black line reproduces the case without QE.

As QE reduces the level of public debt available to investors (panel a), it increases the interest rate gap Δ (panel d). Inside the liquidity trap, this has no effect on real variables and only changes the composition of assets held by investors. When exit is early, QE has therefore little real effects. The only impact happens as the economy exits the trap: the surprise increase in the price level, which inflates away the value of money held by investors, has a stronger effect with QE since investors hold more money, leading to a slightly lower capital stock at exit.¹⁹ A late exit, by contrast, has a substantial impact on the economy. Absent QE or with an early exit, the interest rate would increase to slow down investment as the economy reverts, bringing the capital stock close to its steady state level. Since QE deprives the economy of bonds, the interest rate stays stuck at the ZLB (panel b) and the investment boom goes unhampered (panel g). Instead, adjustment comes again from the price level, leading to stronger inflation than with an early exit (panel f). This is only temporary.²⁰ As agents expect QE to be eventually reversed, the price level slowly decreases back to its steady state level, increasing the real rate and hurting capital and output. The interest rate only leaves the ZLB when QE is finally undone. Therefore, with a late exit from QE, the economy overheats, before plummeting again, instead of quickly going back to normal.

Public Debt and Capital Getting out of the liquidity trap through a higher public supply of liquidity, while leading to better consumption smoothing thanks to a higher interest rate, has an ambiguous impact on capital accumulation and output in the general case. This impact depends on the level of liquidity \bar{l} that prevails at the exit of the liquidity trap. Indeed, out of the ZLB, the interest rate starts to increase, and the net position of investors \bar{l} determines the effect of a higher r on capital accumulation. In our benchmark case where investors are initially in autarky, the increase in public debt makes them net creditors as they exit the ZLB,

¹⁹With nominal instead of real bonds, QE with an early exit would have no effect at all.

²⁰Auerbach and Obstfeld (2005) consider a one-time open-market operation which is never reversed. Their economy experiences an increase in inflation and seigniorage, which allows to decrease distortionary taxes. In their representative individual framework, they show that welfare increases with this experiment.

which typically leads to an increase in the capital stock towards its efficient level.

We show in Section 3.2 of the Online Appendix that the liquidity injection can also lead to a lower capital stock out of the liquidity trap, especially if investors are initially net debtors and the deleveraging shock is not too large.

Welfare and Pareto Efficiency While an increase in liquidity makes the economy *converge* to a Pareto-efficient steady state, the whole equilibrium including transition dynamics is not in general a Pareto equilibrium. The higher interest rate indeed initially hurts borrowers and temporarily decreases investment even lower than its liquidity trap level. In addition, the new steady state does not always Pareto-*improve* on the initial one, as in the case mentioned above where capital decreases in the long run, which lowers wages and hurts workers.²¹

Addressing these two problems requires many additional policy instruments. Section 4.3 of the Online Appendix shows how three additional taxes/subsidies make it possible for the policy maker to implement a Pareto-efficient equilibrium path (including the transitory dynamics) that Pareto-improves on the initial liquidity trap.

Credit Easing Our model does not account for the fact that QE sometimes goes hand-in-hand with credit easing aimed at improving credit conditions for the private sector, which can alleviate the effect of deleveraging. Credit easing would consist in the government issuing new debt dl^g to lend an amount $(d\phi)\alpha y$ to I -investors above the limit of their borrowing constraint, effectively relaxing this constraint. As $dl^g = (d\phi)\alpha y$, the government net debt does not change and stays equal to l^g . Credit easing can therefore be effective in getting out of the liquidity trap, as it helps re-leveraging investors after a deleveraging shock. Similarly to public debt issuance, the interest rate gap is closed by increasing the shadow interest rate.

4.2 Lowering the Effective Real Interest Rate

Increasing liquidity closes the interest rate gap by increasing the shadow interest rate. The alternative is to decrease the effective rate. This could be done by increasing expected inflation through an increase in θ . This is a natural solution mentioned in the literature on the liquidity

²¹A potential third issue is that reducing the capital stock can be undesirable in its own respect if there are external growth spillovers for example.

trap (e.g., Krugman (1998)). Alternatively, there could be a negative nominal interest rate. Suppose that cash is replaced by Central Bank digital money, on which a negative interest rate can be charged. There would then be no ZLB on the nominal interest rate and we could have $i < 1$.²²

A lower effective rate can make the economy exit the ZLB, restoring a higher level of capital and output. Workers would then be better off thanks to higher wages, but the lower interest rate would impair consumption smoothing. Exiting the ZLB by reducing the effective real interest rate drives out monetary liquidity without providing alternative liquidity and solving the underlying asset scarcity problem. In some cases, when investors are net debtors, the lower interest rate can even lead to capital over-accumulation.

A timidity trap If the effective rate is not lowered completely to the shadow rate, this has an ambiguous effect on capital and output. Consider a slightly higher inflation θ . Besides a decline in the effective real interest rate, this also increases the cost of holding money. This negative effect dominates when money holdings are large. In fact, we show in Section 3.3 of the Online Appendix that, in a liquidity trap, a higher inflation rate θ has a negative effect on capital at the margin if both ϕ and θ are low, as they induce high money holdings. However, an increase in inflation that is large enough to drive the economy out of the liquidity trap has a positive effect on the capital stock.²³

4.3 Fiscal Policy

In the baseline policy regime, the fiscal deficit τ^w adjusts to the policy mix (θ, \bar{l}^g) . However, the fiscal deficit could also become the dominant policy parameter. In that case, either θ or \bar{l}^g needs to adjust. However, we show here that the adjustment to a fiscal deficit cannot come from public debt in a liquidity trap. If a permanent fiscal deficit can be financed at all, it can only be through the inflation tax, so that the analysis of 4.2 applies. Aggressive fiscal policy can therefore bring the economy out of the liquidity trap and stimulate the economy (although it leads to an inefficient equilibrium). While this prediction is reminiscent of the standard

²²A liquidity trap with a negative interest rate is a situation currently observed in several countries.

²³A similar analysis holds for small decreases in the nominal interest rate i .

effect of fiscal policy at the zero-lower-bound, the channel here is not an aggregate demand channel whereby fiscal policy creates inflation expectations that stimulate private consumption. Inflation increases to guarantee the solvency of the government, and higher inflation stimulates the investment capacities of investors.

Fiscal Deficit and Inflation Tax In the normal steady state, government debt \bar{l}^g can accommodate the fiscal deficit τ^w , as apparent through the government budget constraint (27). However, in the liquidity trap, \bar{l}^g becomes irrelevant, and only a higher inflation θ can accommodate a higher fiscal deficit. To see this, aggregate the demand for bonds and money to get:²⁴

$$\theta(m + \bar{l}y) = \theta s + (1 - \alpha)y \quad (28)$$

where $s = m^S + \bar{l}y$ is the aggregate demand for saving by investors as defined earlier. We can then rewrite the government budget constraint (27) in the liquidity trap as

$$(\theta - 1)[(1 - \alpha)/\theta + s/y - \bar{l}^w] = \frac{\tau^w}{y}, \quad (29)$$

where s/y is given by (25). In the liquidity trap, the level of government debt \bar{l}^g no longer appears in the government budget constraint. Instead $m + \bar{l}y$ adjusts through m whenever \bar{l}^g changes, because at the given liquidity trap interest rate the private sector is not willing to hold more government liabilities. Therefore, the composition of government liabilities changes without affecting its total amount, leaving the government budget constraint unaffected since in a liquidity trap, the composition of government liabilities does not matter.

This implies that with an increase in the fiscal deficit τ^w , only θ can adjust to maintain solvency by creating an inflation tax. In that context, a permanent increase in the fiscal deficit is necessarily inflationary. The fiscal deficit may therefore be effective in helping the economy getting out the liquidity trap, but only because it requires higher inflation-driven seigniorage. Government spending, by increasing the deficit, would have similar effects.

An Inflation Tax Laffer Curve There is, however, a limit to the fiscal income that can be generated through inflation. Indeed, a higher inflation also decreases the demand for gov-

²⁴Equation (28) follows from (16) taken in a liquidity trap steady state, together with the definition of s .

ernment assets, which reduces seigniorage. We can show, by differentiating the LHS of (29), that there is a Laffer curve for inflation, where the maximum inflation tax is reached for $\theta = [(1 - \alpha) + \alpha(1 - \phi)\beta] / (\alpha\phi + \bar{l}^w)$. Beyond that point, a permanently higher fiscal deficit is unsustainable. In a liquidity trap, there is a limit to the use of fiscal policy, as there is a limit to both the issuance of public liabilities and to the inflation tax.

5 Extensions

Workers' Deleveraging We have considered so far a deleveraging shock on investors, modeled as a decline in ϕ . Likewise, a deleveraging shock on workers can be modeled by a drop in \bar{l} , coming from a drop in \bar{l}^w .²⁵ Such a shock limits the economy's supply of assets and has a similar effect on the interest rate r as a deleveraging shock on investors, as can be seen from Equation (23). Workers' deleveraging can therefore also lead to the zero lower bound. This is illustrated in the right panel of Figure 5, where a decrease in \bar{l} makes the economy switch from C , a normal steady state, to T , a liquidity trap, through a fall in r .

However, once the economy is in a liquidity trap, changes in \bar{l} have no effect. Indeed, since the interest rate cannot adjust in a liquidity trap, the net demand for assets s is constant, so any decrease in the supply of assets to investors through \bar{l} is matched by an increase through m^S . As before, higher real holdings of money obtain through a downward shift in the path of prices. The key difference between a deleveraging shock on workers and on investors is that the former affects the supply of assets to investors, while the latter affects their net *demand* for assets. Both are fully accommodated by an adjustment in real money holdings, but only a change in demand actually changes the asset holdings of investors, which is the source of disinvestment.

In fact, we show in Section 3.2 of the Online Appendix that, as investors become net debtors following workers' deleveraging ($\bar{l} < 0$), this shock has a positive effect on capital outside the liquidity trap.²⁶ This is illustrated in the left panel of Figure 5. When switching from the

²⁵Note that this shock might imply a positive net position of workers ($\bar{l}^w < 0$). This is consistent with a high proportion of wealthy hand-to-mouth households, that is, households who own sizeable amounts of illiquid assets (like retirement accounts) but hold little liquid assets, as documented by Kaplan et al. (2014).

²⁶In the normal steady state, as shown by Equations (22) and (23), changing the net liquidity position \bar{l} has two effects. A lower \bar{l} has a positive effect on investment as liquidity has net cost $1/r - \beta > 0$. It also

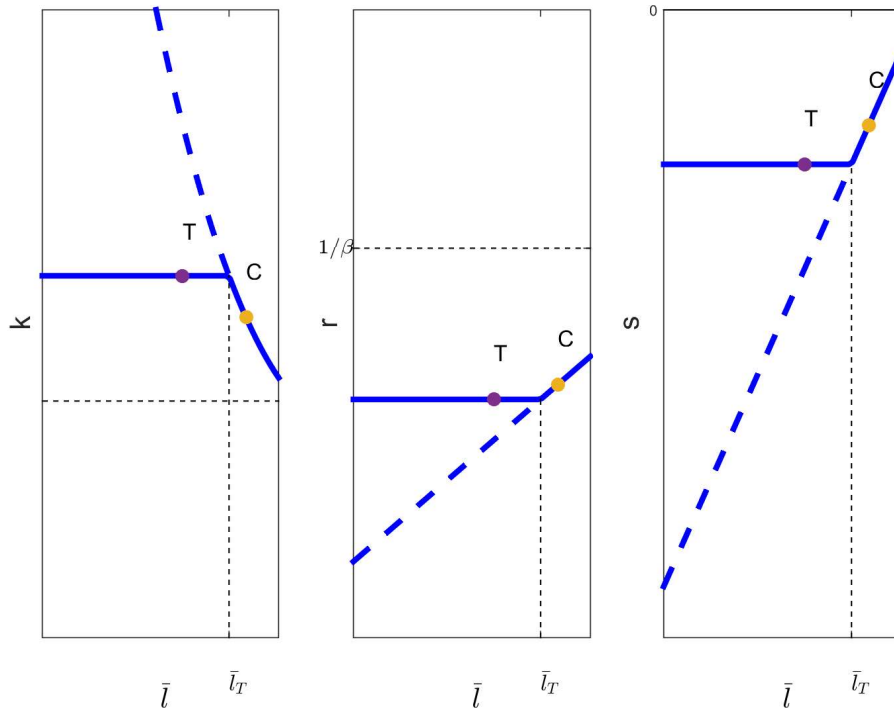


Figure 5: Steady states - Comparative statics w.r.t. \bar{l}

normal steady state C to the liquidity trap T , the economy experiences an increase in the capital stock. However, workers' deleveraging does not affect the long-run capital stock in the liquidity trap.

Thus, in the case where investors are initially in autarky, only an investors' deleveraging that brings the economy to a liquidity trap can decrease capital in the long run.

Bubbles The existing literature has long shown that rational bubbles can obtain in environments with low enough real interest rates.²⁷ In our framework with scarce assets, bubbles can provide additional saving instruments to accommodate the demand for assets by S-investors. A bubble, when it emerges, provides enough liquidity to exit the ZLB. But, as we will show, it also constrains the real interest rate and prevents the natural equilibrium adjustment.

Consider an infinitely-lived asset in fixed unitary supply with no intrinsic value—a bubble. Denote z_t its relative price in terms of consumption goods. The real return of the bubble as

increases the price of liquidity, which either decreases the resources of investors if they are net creditors ($\bar{l} > 0$), or increases them if they are net debtors ($\bar{l} < 0$). Since we assume that investors are net debtors, both effects are positive.

²⁷See Samuelson (1958), Tirole (1985), and more recently Martin and Ventura (2012).

of time t is z_{t+1}/z_t . For the bubble to be traded, this rate of return must be equal to the real interest rate: $z_{t+1}/z_t = r_{t+1}$. With r_{t+1} different from 1, the bubble would either asymptotically disappear or diverge to an infinite value. Then, a bubbly steady state necessarily has a zero *real* interest rate: $r = 1$. With positive long run inflation, $1 > 1/\theta$ so the bubble strictly dominates money as a saving instrument. Therefore, S-investors would hold the bubble and would not hold money.²⁸ In the case of autarkic investors, such a bubbly steady state is described by:

$$z = \alpha[(1 - \phi)\beta - \phi]y \quad (30)$$

$$k = \beta\alpha y - (1 - \beta)z \quad (31)$$

where (30) is the Euler equation of savers and (31) the aggregate budget constraint of investors. As can be seen from equations (19) and (21), the bubbly steady state is formally equivalent to a liquidity trap steady state with $m^S = z$ and $\theta = 1$. The bubble plays the same role as investor-held money in the liquidity trap, but offers a higher real return.

We show formally in Section 3.5 of the Online Appendix that a bubble can indeed help the economy exit the liquidity trap if $\theta > 1$. The bubble raises the nominal interest rate from $i = 1$ to $i = \theta$. S-investors then substitute the bubble for money in their portfolio. For a given money supply, this also reflate the economy as the price level increases to accommodate the lower money demand.

However, the bubbly steady state is qualitatively similar to a liquidity trap. As with money, holding the bubble takes out resources from investment and output is lower in the bubbly equilibrium than in the normal steady state. In the intermediate case where $\phi_T \leq \phi < \phi_B$, where ϕ_B is a threshold value defined in the Online Appendix, a bubble prevents the downward interest rate adjustment that would restore the normal level of capital and output. In the case of low leverage $\phi < \phi_T$, bubbles increase the real interest rate, which may or may not increase capital and output compared to the liquidity trap. This is similar to the ambiguous effect of inflation described in Section 4.2.

²⁸With negative long-run inflation, bubbles would be dominated by money and could never arise in equilibrium.

Preference and Growth Shocks In the existing literature, the shock that brings the economy to the ZLB is often assumed to be an increase in the factor of time preference. This shock, by increasing the agents' propensity to save, has a negative effect on the interest rate. A reduction in the average growth rate of productivity has also been put forward as an explanation for the secular decrease in the interest rate and for hitting the ZLB. In fact, in an infinite-horizon model, the effect of a growth slowdown is isomorphic to an increase in the factor of time preference. We therefore restrict our analysis to the latter. We find that a permanent increase in β (alternatively, a permanent fall in steady-state growth), cannot generate a fall in the investment rate when the economy falls into a liquidity trap.

Indeed, we show in Section 3.6 of the Online Appendix that an increase in β makes the long-run interest rate fall, and eventually hit the ZLB. In both the normal and liquidity-trap steady states, an increase in β increases the investors' propensity to save, which increases the capital stock in the long run. As a result, whereas an increase in β can explain the emergence of a liquidity trap, it cannot explain the slowdown in investment. In the presence of trend growth, the same conclusions would hold in case of a growth slowdown. In particular, with lower trend growth, less investment is required to keep the capital stock on its trend. Therefore a given amount of saving leads to an upward shift in the capital intensity of production, and hence in the investment rate.

Financial Intermediation In the benchmark model, money is modeled as *outside money* directly supplied by the government. However, in practice, cash holdings usually take the form of deposits, which are a liability of banks, and could in principle be intermediated to capital investment. We show in Section 3.8 of the Online Appendix that this is not the case. At the ZLB, banks are unable to channel deposits to credit-constrained I-investors for the same reason that savers are unable to do so in the benchmark model. Instead, banks increase their excess reserves at the central bank.

Inefficient saving technology The benchmark model assumes that bonds and money are the only available saving instruments. In Section 3.9 of the Online Appendix, we extend the model by allowing for an inefficient storage technology, with a rate of return $\sigma \in (\theta^{-1}, \beta^{-1})$ and concave installation costs. This technology starts being used by savers when the interest rate

falls down to σ . Then, a moderate deleveraging shock reallocates saving to the storage technology, which crowds out “good” capital even in the normal equilibrium. This reallocative effect is similar to the one studied by Buera and Nicolini (2016). With a large enough deleveraging shock, the economy falls into the liquidity trap, the use of inefficient storage is pinned down by the real rate of interest $1/\theta$, and higher money holdings crowd out capital as in the benchmark model. One difference with the benchmark model is that the shadow rate now has a strictly positive lower bound as ϕ goes to 0, since the storage technology prevents a complete collapse of intertemporal trade, arguably a more realistic feature.

Idiosyncratic Uncertainty The benchmark model with deterministic transitions between saving and investing phases can be easily extended to stochastic transitions. We consider in Section 3.10 of the Online Appendix a 2-state Markov process where an investor with no investment opportunity at time $t - 1$ receives an investment opportunity at time t with probability $\omega \in (0, 1]$; while an investor with an investment opportunity at time $t - 1$ receives no investment opportunity at time t . We then show that results from the benchmark model extend to the case of idiosyncratic uncertainty.

Partial Capital Depreciation In most of the analysis, we assumed full capital depreciation. In Section 3.7 of the Online Appendix, we allow the depreciation rate of capital to be lower than one, so that capital depreciates only partially from period to period. For consistency, we focus on the case where investors are autarkic or net debtors ($\bar{l} \leq 0$). All our results generalize provided some mild condition on \bar{l} .

Nominal Government Bonds We have assumed so far that government bonds were issued in real terms. In reality though, a large share of government bonds are nominal. In our deterministic setting, assuming that bonds are nominal instead of real is innocuous and all our results generalize to nominal bonds (see Section 3.11 of the Online Appendix).

6 Conclusions

The liquidity trap that followed the Global Financial Crisis has been more persistent than expected. The liquidity trap has lasted even longer in Japan. In most countries, this has been accompanied by a slower than expected recovery and a surprising accumulation of money holdings. In this paper, we explored the long-term implications of a liquidity trap and found that a deleveraging shock may lead to a negative relationship between money and capital. We analyzed policies in a liquidity trap by examining their impact on the wedge between the effective real interest rate and the shadow rate.

While most of our analysis is conducted in a stylized benchmark model, the main mechanism is robust to many extensions. The extensions considered in the paper include bubbles, partial capital depreciation, idiosyncratic uncertainty, nominal bonds, or introducing an alternative saving technology or financial intermediaries. While our theoretical results are derived with a permanent deleveraging shock for investors, we show in simulations that they also obtain in the medium run for persistent shocks with nominal rigidities.

According to our results, long-term output declines in a liquidity trap only with investors' deleveraging. Other positive shocks to saving, like workers' deleveraging or an increase in the discount rate, may also lead to a liquidity trap, but they do not depress output in the long run. Therefore it is crucial to determine the factors that have led to a liquidity trap. Interestingly, Galí et al. (2012) suggest that financial shocks have played a key role in the slow recovery.

Overall, our approach is complementary to Keynesian analyses that stress the role of insufficient demand in a liquidity trap. While they describe a situation of negative output gap when the adjustment of prices is hampered by nominal rigidities, we show that low investment demand leads to lower potential output even after prices have fully adjusted. Our framework also enables to examine policies that are complementary to more standard demand management. In this context, we find that quantitative easing is ineffective at the ZLB and can deepen and possibly lengthen the liquidity trap. We also argue that it may be better to increase the shadow rate than decrease the effective real interest rate.

A Proofs

We establish first the following Lemma:

Lemma 1 *The normal and liquidity trap steady states are characterized as follows:*

(i) *In a normal steady state,*

$$r^* = \frac{\alpha\phi + \bar{l}}{\beta\alpha(1 - \phi)}, \quad k^* = [\beta\alpha - \bar{l}(1/r^* - \beta)]^{\frac{1}{1-\alpha}}, \quad m^{S*} = 0.$$

(ii) *In a liquidity-trap steady state,*

$$\hat{r} = 1/\theta, \quad \hat{k} = \left(\frac{\beta^2 + \phi(\theta^2 - \beta^2)}{\theta/\alpha} \right)^{\frac{1}{1-\alpha}}, \quad \hat{m}^S = \alpha \left[(1 - \phi)\frac{\beta}{\theta} - \phi - \bar{l}/\alpha \right] \hat{k}^\alpha.$$

Proof. In a steady state, the money market equilibrium implies that $P_{t+1}/P_t = \theta$. As a result, $i = r\theta$.

In a steady state with $i^* > 1$, (14) and (15) are satisfied with $M^S = 0$. Equation (14) taken at the steady state gives r^* . Besides, (15) in the steady state gives:

$$k^*/y^* = \beta\alpha - \bar{l}(1/r^* - \beta)$$

which yields our result for k^* . This proves result (i).

In a steady state with $i = 1$, (14) and (15) are satisfied with $r = \hat{r} = \theta^{-1}$, which yields $\hat{k}/\hat{y} = \frac{\beta^2 + \phi(\theta^2 - \beta^2)}{\theta\alpha^{-1}}$, from which we derive \hat{k} , and $\hat{m}^S = \alpha \left[(1 - \phi)\frac{\beta}{\theta} - \phi \right] \hat{k}^\alpha - \bar{l}\hat{k}^\alpha$. This proves result (ii). ■

A.1 Proof of Proposition 1

Consider a normal steady state with $\bar{l} = 0$. According to Lemma 1, $r^* = \phi/[\beta(1 - \phi)]$. We check that $0 < \beta r^* < 1$ as $\phi < \phi_{max}$ and that $i^* = \theta r^* \geq 1$ as $\phi \geq \phi_T$, which insures that the normal steady state exists and is locally constrained. This proves result (i).

If $\phi < \phi_T$, then the steady state without money does not exist, as the implied nominal interest rate i^* would be below one. If there exists a steady state with $i = 1$, then it is a liquidity

trap described by Lemma 1. According to Lemma 1, when $\bar{l} = 0$, $\hat{m}^S = \alpha \left[(1 - \phi) \frac{\beta}{\theta} - \phi \right] \hat{k}^\alpha$, which is strictly positive when $\phi < \phi_T$. Besides, $\hat{r} = \theta$, which implies that $0 < \beta \hat{r} < 1$ under Assumption 1, and $\hat{r} > r^*$ for $\phi < \phi^T$. We also check that $\hat{k} = \left(\frac{\beta^2 + \phi(\theta^2 - \beta^2)}{\theta/\alpha} \right)^{\frac{1}{1-\alpha}} < k^* = (\beta\alpha)^{\frac{1}{1-\alpha}}$ for $\phi < \phi^T$. This proves result (ii). Results (iii) and (iv) derive naturally from Lemma 1.

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